Class XII - MATHEMATICS

Time allowed: 3 Hours Max. Marks: 100

General Instructions

1. All questions are compulsory.
2. This question paper consists of 29 questions divided into three Sections A, B & C
3. Section A comprises of 10 questions of one mark each,
   Section B comprises of 12 questions of four marks each
   Section C comprises of 7 questions of six marks each.
4. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
5. There is no overall choice. However, internal choice has been provided in 04 questions of four marks each and 02 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
6. Use of calculators is not permitted.

SECTION A

(each question carries 1 mark)

1. If X $= \begin{pmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{pmatrix}$ and X + X' = I then find x for $0<x<\pi/2$.
2. A matrix A of order n has $|A| = 2$ and $|A\,adjA|$ = 8. Find n.
3. Is the matrix ABA' symmetric or skew symmetric provided the matrix B being skew symmetric matrix? Why?
4. Can * be a binary operation defined on N where a*b = a – 2b? Why?
5. Find dy/dx , given y = tanx$^\circ$.
6. Evaluate: $\int e^{3\log x} x^4 dx$
7. Evaluate $\int_{-\pi/2}^{\pi/2} \sin^5 x \cos^4 x dx$
8. Find the differential equation of all non-vertical lines in a plane passing through the origin.
9. Find the projection of $\bar{a} = 2\hat{i} - 3\hat{j} + \hat{k}$ on $\bar{b} = \hat{i} + 2\hat{j} + \hat{k}$
10. Find any one value of Sec $^{-1}$ ($\sqrt{2}$) other than the principal branch.

SECTION B

(each question carries 4 marks)

11. If $f : R - \{-1\} \to R - \{1\}$ be defined as $f(x) = \dfrac{x-1}{x+1}$ and $g : R - \{1\} \to R - \{-1\}$ be defined $g(x) = \dfrac{1+x}{1-x}$ .Show that $g \circ f = I_A$ and $f \circ g = I_B$ where A = $R - \{-1\}$, and B = $R - \{1\}$
12. If $\sin^{-1}(1-x) = 2\sin^{-1} x + \pi/2$ then solve for x. 'OR'
    If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$; *show that* $x^2 + y^2 + z^2 + 2xyz = 1$
13. Show that the differential equation $2ye^{\frac{x}{y}}dx + (y - 2xe^{\frac{x}{y}})dy = 0$ is homogeneous and find its particular solution when x = 0 and y = 1.

14. Find a point on the parabola $y = (x - 3)^2$, where the tangent is parallel to the chord joining (3, 0) and (4, 1).

15. If $x = \sin^3 t / \sqrt{\cos 2t}$ and $y = \cos^3 t / \sqrt{\cos 2t}$ then find $\frac{dy}{dx}$ without eliminating the parameter 'OR'

Find the differential coefficient of $\tan^{-1}\frac{2x\sqrt{1-x^2}}{1-2x^2}$ w.r.t $\sec^{-1}\frac{1}{2x^2-1}$ at $x = \frac{1}{2}$

16. If the function $f(x) = \begin{cases} x^2 + ax + b, & 0 \le x < 2 \\ 3x + 2, & 2 \le x \le 4 \\ 2ax + 5b & 2 < x \le 8 \end{cases}$ is continuous in [0,8] find the value of a and b.

17. If $\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$, then find the value of $\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c}$ where $(p \ne a,\ q \ne b,\ r \ne c)$.

18. Using the method of integration, find the area of the region bounded by the curves $x^2 + y^2 = 4$ *and* $y^2 = 3x$

19. Evaluate $\int_0^{\pi/2} \frac{\cos x}{1 + \sin x + \cos x} dx$. 'OR'

Evaluate $\int \frac{\sqrt{x^2+1}[\log(x^2+1) - 2\log x]}{x^4} dx$

20. If $\hat{a}, \hat{b}$ *and* $\hat{c}$ *are* any three vectors then show that $(1 + |\hat{a}|^2)(1 + |\hat{b}|^2) = (1 - \hat{a}.\hat{b})^2 + (\hat{a} + \hat{b} + \hat{a} x \hat{b})^2$

'OR'

If $\hat{a} and \hat{b}$ are unit vectors inclined at an angle θ, then prove that $\sin\frac{\theta}{2} = \frac{1}{2}|\hat{a} - \hat{b}|$

21. If $y = e^{ax}\sin bx$. Show that $\frac{d^2y}{dx^2} - 2a\frac{dy}{dx} + y(a^2 + b^2) = 0$

22. A speaks truth in 70 percent of the cases and B in 90 percent of the cases. In what percentage of cases, are they likely to contradict each other in stating the same fact.

SECTION C

(each question carries 6 marks)

23. A wire of length 35m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the length of the two pieces so that the combined area of the square and the circle is minimum? 'OR'

Show that the area of a right-angled triangle of a given hypotenuse is maximum when the triangle is isosceles.

24. Prove that $\int_0^{\frac{\pi}{2}} \frac{x\sin x\cos x}{\cos^4 x + \sin^4 x} dx = \frac{\pi^2}{16}$

25. Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and hence solve the system of equations

$x - y + z = 4$, $x - 2y - 2z = 9$ and $2x + y + 3z = 1$.

26. By examining the chest X- rays, the probability that T.B is detected when a person actually suffering is o.99. The probability that the doctor diagnoses incorrectly that a person has T.B on the basis of X-ray is 0.001. In a certain city 1 in 1000 person suffers from T.B. A person is selected at random and is diagnosed

to have T.B. what is the chance that he actually has T.B.?

27. Find the distance of point P (3,-1, 1) from the plane 2x-y-z =1 measured along the line $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{0}$.

'OR'

A line makes angles α, β, γ and δ with the diagonals of a cube, prove that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = 4/3$

28. Find the shortest between the lines $\bar{r} = (6+s)\hat{i} + (2-2s)\hat{j} + (2s+2)\hat{k}$ and

$$\bar{r} = -4\hat{i} - \hat{k} + t(3\hat{i} - 2\hat{j} - 2\hat{k})$$

29. Determine graphically the minimum value of the objective function: Z=-50 x+20y

Subject to the constraints $2x-y \geq -5$; $3x + y \geq 3$; $2x-y \leq 12$; $x \geq 0$; $y \geq 0$